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TECHNOLOGY IN MATHEMATICS TEACHING: NO USE AT ANY PRICE

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More than 25 years of experience in mathematics teaching lead the author to the conclusion: Technology in Mathematics Teaching: Less is More. The international assessments TIMMS and PISA had a big impact on the mathematics education at school in Germany. The paradigm changed. "Reduce the curriculum to basics. Reduce the predominance of teaching the formal skills therefore improve the understanding." (Baptist & Raab, 2007). Mathematics should be linked to real world problems. At school the focus is now on modelling using technology. Now we can observe the results of this development at university. Written exams show terrible elementary mistakes we rarely have seen before. And even overall fairly good students demonstrate these mistakes. We will give examples and little statistics of some of these problems. We also compare the level of books for mathematics used at German schools over years.

Keywords: Elementary mistakes, Underestimation of exercising

LOOK BACK TO THE NINETIES AND WHAT HAVE CHANGED?

I attended the conferences ICTMT 3 (1997) at Koblenz and ICTMT 5 (2001) at Klagenfurt. There I emphasized that technology in mathematics teaching at university enables to utilize the total new possibilities of virtual experiments in mathematics. I hoped that technology gives chances to get students more active. The aim was to enhance the mathematics understanding of the students. Indeed, students became more active, but the effect was much smaller than I expected.

What has changed since the nineties?

In the nineties and in the early 2000er years we observed a lack between the mathematical performance of the university freshmen and their expected competencies. Now the gap becomes much bigger between how the freshmen perform and how they should do perform. (Schwenk & Kalus, 2012; Schwenk & Berger, 2006). In the nineties the students had been much better trained at school in calculating without electronic tools than nowadays. Technology at school was limited to simple pocket calculator. Today computer algebra calculators are widely spread and widely used at school.

The international assessments TIMMS and PISA had a big impact on the mathematics education at school in Germany. The so called "SINUS Transfer" project was established in 2007 (Baptist & Raab, 2007). The SINUS experts suggested:

- Reduce the content to the basics,
- reduce teaching formal skills,
- avoid plantations of exercises,
- improve the understanding,
- model real world problems using technology,
- avoid drilling exercises by marching in step,
- promote individual learning,
- teach as moderator of students who follow their own responsibility.

One of the objectives was to point out that mathematics is useful and needed in everyday life. For that a frequent “modelling of real world problems” entered the schools. But here we have to object that nobody is able to model a problem without knowing anything about the field to be modelled. As a consequence instead of real world problems, exercises dressed up in a carnival costume appear. In addition to that the modelling is often already done by the authors of the exercises, which means that the students do not really need to model. An examples is given in figure 1, see below.

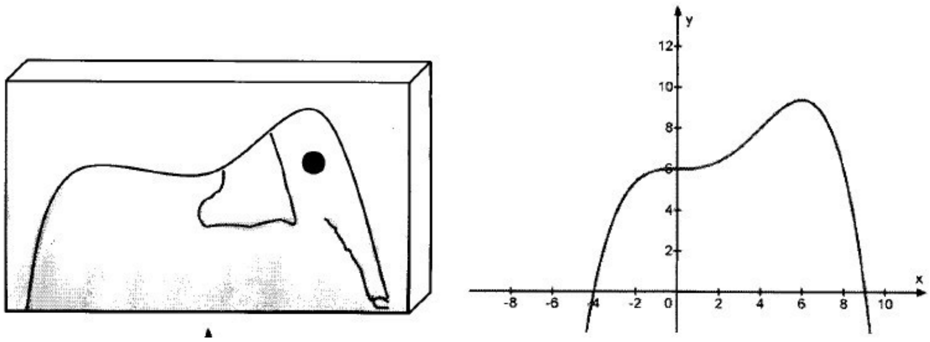

Figure 1: A wooden elephant modelled entirely by polynomial of 4th order.

The story of this problem is to get the need of wood for the elephant. The contour is determined by a given polynomial of 4th order. But no producer of toys will model an elephant with a single polynomial. A similar example is the modelling of an artificial island by a given polynomial of 3rd order (Anonymous, 2013). There are many other examples dealing with unrealistic situations like the trajectory of flying birds following a straight line. Then modelling the trajectory by parameterisations with vectors should give possible meeting points of the birds. It is not convincing that these kinds of problems far away from reality will increase the motivation of students. Modelling focuses often to the process of generating the solution, then calculations are often regarded as secondary and are delegated to the pocket calculator.

In parallel computer algebra pocket calculators (CAS calculator) arose in schools. The use of the CAS calculator was forced by the supervisory school authorities. For some years the school authorities are offering two versions of the centralized final A-level examination (Abitur) in mathematics so that the schools can choose the CAS version or the normal version. In order to prepare the students for the CAS-examination CAS calculator are intensively used at school.

CONSEQUENCES: EXAMPLES

In the following I give some examples of severe mistakes I found in written examinations of a mathematics course for electrical engineering students. Mostly these severe mistakes are known for a long time. But what is new, is the frequent occurrence of them. One of my prominent examples of basic deficits in freshmen mathematical skills was that 4 of 52 Students of mine in a written test without using a pocket calculator stopped at $\sqrt{9}$ and did not simplify it to 3. The concept of a square root seemed to be reduced to the corresponding button of the calculator. Square root was strictly linked to the use of pocket calculator. Of course, all the students knew the value of $\sqrt{9}$, but it seemed that the automatism had gone. This has been my starting point of further investigations and of statistically documenting the mistakes.

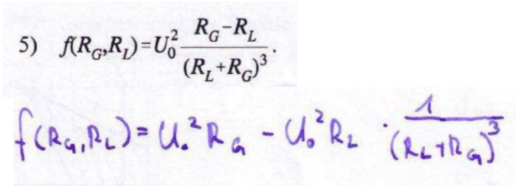

Figure 2: Transforming a fraction into a product

The example of figure 2 shows a missing ability transforming a fraction into a product. The problem was: Build the partial derivative with respect to R_G. 20% of all participants showed this mistake (see figure 2). But only a part of the participants tried to avoid the quotient rule by transforming the term into a product. Even though only this part of all participants could fall into this trap after all 20% (7 of 35) of all participants did not bear in mind that a fraction bar has an effect like brackets.

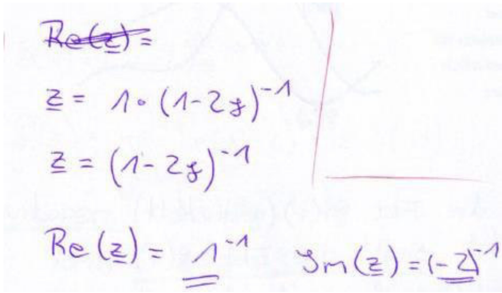

Figure 3: Inverse of a sum

The next example is taken from a quick test (see figure 3). Pocket calculators were not allowed. The problem was to identify the real and the imaginary part of the given complex number. 29% of all participants built the sum of the single inverses instead of looking for the inverse of the sum.

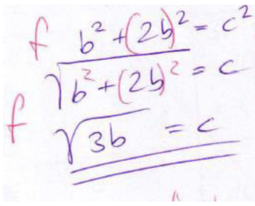

Figure 4: Square of a product

The problem in figure 4 was to calculate the length of the missing edge of a right triangle. One leg of the triangle was given by b and the hypotenuse by $2b$. 39 % that means 14 of 36 of all participants failed in squaring the product. By the way the figure 3 shows other nice mistakes, but these were of singular occurrence.

LOOKING FOR REASONS

An evaluation of the final marks points out that even overall fairly good students do these kind of simple severe mistakes. In general most students have the knowledge to do it correctly. However, if students are confronted with a single mathematical operation they will be able to solve it correctly. But when students have to split their attention to more than one point they get confused. Elsbeth Stern, an expert in cognitive psychology at the ETH Zürich, Switzerland, emphasises the role of practise: “A person, who is not experienced in reading, has to transform every letter into a phoneme and has arduously to construct a word out of it. RAM capacity is occupied that is lost for understanding the content.” (Stern, 2006, translated by Schwenk). Many of our engineering freshmen have comparable difficulties. They are not experienced in reading formulas, they spell the mathematical expressions. Thus RAM capacity is bound that goes lost to capture the meaning and the structure of the expression. Like reading, doing the basics in mathematics has to be automated. Automation releases RAM capacity, this is needed for the creative process of understanding and problem solving and saving new information.

If during mathematics education a CAS pocket calculator is used to early or/and too much this will lead to less routine of elementary mathematical skills. Competencies will not reach an automated level. Spitzer says "The brain permanently adapts itself to its use." (Spitzer, 2012) This means: If students are used to computer algebra systems, they do not really know how to deal with fractions, because they can restrict to copy fractions in an optical way to the CAS computer. Then the CAS computer will analyse the expression not the students. Finally the student's concept of fractions fades. If students use CASystems for differentiations, the computer will differentiate not the students. It is the computer that analyses the function and chooses the right differentiation rule, but not the students. The result is that the students get a lack of understanding of formulas.

Pocket calculators draw graphs of functions and students just copy it. Therefore the concept of functions is reduced to single buttons of the calculator. The difference between the multiplication button and p. e. the cos-button is disappearing. I observed mistakes where applying functions is considered as multiplications. For example the product rule was used for the differentiation of $\cos(2x)$ with one factor to be cos and the second factor to be $2x$. There are many students with a poor developed concept of functions.

An intensive use of computers obstructs students to feel responsible themselves for their knowledge. So it happens, that finally they have linked any square root to the corresponding button und do not simplify simple square roots by their own (see above).

A colleague of mine reported her students felt disadvantaged because of their poor abilities they got from school. At school they had used intensively a CAS calculator. They said they did not understand their input to the calculator but got at school the best mark. They always solved equations by the solve-button, functions were plotted immediately and the graphs were not discussed.

The same change of paradigm of teaching mathematics at German schools has started earlier at the end to the eighties in the Netherlands. Krieg, Verhulst and Walcher reported about a protest of students in the Netherlands against the low level of mathematics teaching at school. The students expressed their alarm in a public letter to the minister of education Maria van der Hoeven, signed by 10 000 students. This letter has become popular under the motto "Lieve Maria". (Krieg, Verhulst, & Walcher, 2008)

As we pointed out at a SEFI conference we also must take into account that student' life and our daily life in general have changed. There is a flood of information. Looking for solutions is now a quick and impatient online search. New devices like digital cameras, mobile phones etc. must be self-explanatory without long instructions. The young generation can be regarded as natives of our digital world, while the older generation, born in the non-digital world, are immigrants. The natives of the digital world are experts in the trial-and-error method and in looking for answers just by one click. But they are not trained for a time consuming, systematic and deductive acquisition of knowledge. They are not well trained to follow formal rules of either a natural language or a mathematical language. (Schwenk & Kalus, 2010)

CHANGE OF PARADIGM OF TEACHING: CRITICISM

Behind the modern way of teaching mathematics lies an underestimation of the importance of the formal calculus and of exercising. Spitzer says (as cited in Siebert, 2003) "Train at first simple but fundamental examples ". Exercising simply problems is a prerequisite for the following abstract level. "Often it is not clearly distinguished between the role of calculating in the context of solving real world problems and the role of learning calculating itself in the context of learning mathematics." (Schröder, 2015, translated by Schwenk).

A Comparison of German school books (7. years of school) demonstrates the dilemma.

4. **a)** $[(-6)+(-8)]\cdot(-7)$ **e)** $\frac{-24}{-3}+5\cdot(-6-11)$ **i)** $\dfrac{-3\cdot[(+18)+(-6)]}{27:(-3)}$

b) $(-8-4)\cdot(6-2)\cdot(-\frac{2}{3})$ **f)** $\dfrac{[-29-(-15)]\cdot(-5+16)}{-6-1}$ **j)** $\dfrac{(3-8)\cdot(-25+12)}{[3-(-6)]\cdot(-\frac{5}{18})}$

c) $[(-9)+6]:[-3-(-8)]$ **g)** $[(-11)-(+14)]:(-\frac{1}{5})$

d) $[11-17-(13-2)]\cdot(-8+2)$ **h)** $(-\frac{1}{2}+4{,}8+\frac{7}{5}):(-\frac{3}{4}:2)$ **k)** $\dfrac{(2{,}6-4{,}6)\cdot(-\frac{1}{2})-1}{\frac{3}{7}:(-\frac{18}{35})-2\cdot(-\frac{7}{24})}$

Figure 5: German school book used in 1992-1999

5 Berechne.
a) $\frac{2}{7}\cdot\left(-\frac{3}{14}\right):\left(-\frac{2}{5}\right)$ b) $\left(-\frac{4}{3}:\frac{8}{9}\right):\left(-\frac{3}{4}\right)$ c) $\frac{3}{2}:\left(-\frac{9}{4}\right):\frac{8}{15}$ d) $\left(-\frac{1}{4}\right)^2:\left(-\frac{7}{8}\right)$

6 Schreibe zunächst als Term und berechne.
a) Dividiere die Summe aus −5 und 7 durch $\frac{1}{2}$.
b) Wie groß ist der Quotient aus dem Quadrat von $\frac{1}{3}$ und dem Produkt aus 3 und $-\frac{2}{5}$?
c) Subtrahiere den Quotienten aus $-\frac{3}{8}$ und $\frac{1}{4}$ von $-\frac{3}{2}$.

7 Während eines Hochwassers hat Herr Schneider täglich gemessen, wie weit das Wasser unter der Kante der Türschwelle seines Hauses steht: −12 cm; −14 cm; −11 cm; −17 cm; −8 cm. Berechne, wie hoch das Wasser durchschnittlich unter der Schwelle stand.

Figure 6: A German schoolbook used in 2006-2010

The figures 5 and 6 show each the most complicated exercises dealing with fractions of rational numbers. Even though the books are written in German language, it can be easily checked that the problems of the newer book are simpler. The style of the book got nicer. We see more pictures. But I think that practising more complicated fractions of numbers is a good exercise for later algebraic terms with variables, and it is a good exercise to find the appropriate order of the single calculations. Thus the literacy of formulas will be enhanced. If these calculations are delegated to a pocket calculator or even to a computer algebra calculator students will miss exercising how to analyse the expression. They will not know how to start at which point of the fraction. Finally students will not know that a fraction bar has an effect like a bracket. Simple problems prepare for abstractions of the next higher level.

Complaining poor elementary mathematical skills is widely spread among universities in Germany and in other European countries. It was a topic on several conferences on engineering education I attended like the 17th SEFI (European Society for Engineering Education) Mathematics working Group Seminar 2014 at Dublin (for example (Breen, Carr & Prendergast, 2014)), or the 12.

Workshop Mathematik für Ingenieure 2015 at Hamburg (for examples the presentations of Susanne Bellmer, Kathrin Thiele & Gerhard Wagner or Thomas Schramm). This was also confirmed by private discussions on two further conferences in 2014.

One obstructive of the new teaching methods was to avoid instructing useless formal calculus and to force understanding. But it seems what we get is neither formal calculus nor understanding. It seems that an active discussion between teaching staff at school and university has not taken place up to now.

LIST OF FAILED EXPERIMENTS IN GERMANY

Reforms in the education field are experiments in vivo and a favourite playground of politicians. After every election the politicians can easily demonstrate their change management in reforming education. Here are some examples of the past that failed in Germany.

1974 in Germany the set theory was introduced at the primary school. The aim was to improve the structural thinking. But parents got confused; they had not been able to support their children doing the homework. Finally the set theory disappeared from primary school.

Next reform was a reform of the final years of upper schools. Students should learn based on their own responsibility. In 1972 a modular system of basic and advanced courses was established, pupils freely chose the courses. Just the minimum number of advanced courses was fixed. Learning became an exemplary learning. In the years after they went back step by step on these reforms. In 1987 the government introduced the obligatory courses German, mathematics, and a foreign language. In 1995 the responsible ministry tried to improve the preparation for university (A-Level, Abitur) within German, mathematics, and a foreign language. In 1997 after TIMMS the paradigm of teaching changed as described above, it was the starting of CAS calculator in schools. The last withdraw was in 2014. The German state Baden-Württemberg will ban CAS calculators in the final mathematics exams as from the year 2019.

CONCLUSION: NO USE AT ANY PRICE

The use of CAS calculators and computers should be carefully weighed. It is necessary that the students got definitely well trained in pencil and paper skills. Paper and pencil skills should also be practised during the phases of using the computer. If it is not continuously trained there will be a loss of skills; the less the practice is established, the more the skill vanishes.

One of the most important guiding principles should be: if it is not necessary to use a computer, then it is necessary to use no computer (Bauer, 1988).

In schools there should also be enough tests without any electronic devises in order to reinforce the elementary skills. The students must feel responsible for their own knowledge.

It is important, that the use of computers must not substitute pencil-paper skills. For example the computer could be used as a control device of solutions by hand. Afterwards the computer could be used for some big real world problems.

Our summary at the ICTMT 5 is still valid: The “phase of critical reflection is very important to working with computers. Otherwise there are two dangers using computers: 1. The teachers are too enthusiastic about the ‘nice’ facilities of the computers so that the students might not be able to follow their teaching. 2. The problems of or for the students may be covered. Before using computers, the teacher has to consider how to check afterwards (using operationalized aims) that his aims have been reached.” (Schwenk & Berger, 2002)

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